Eur. Phys. J. A **14**, 265–269 (2002)
DOI 10.1140/epja/i2001-10211-3

The European Physical Journal A

Short Note

Chiral-invariant phase space model

I. Masses of hadrons

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Received: 12 December 2001 / Revised version: 16 May 2002
Communicated by V. Vento

Abstract. The masses of the $SU(3) \times SU(6)$ hadrons are calculated in the chiral-invariant phase space (CHIPS) model as a sum of the mean energies of the quarks at a constant temperature T_{c} with the color-magnetic splitting and the color-electric shift. The masses of hadrons are parametrized by four constants: T_{c}, m_s, E_{CE} and A_{CM}. With the same number of parameters the CHIPS model fits the masses of hadrons better than the classic bag model. The small mass of the d-quark ($m_d = 2.7\,\mathrm{MeV}$) is used to prove that the isotopic shifts of hadrons can be explained by the mass difference between the d- and u-quarks. The dibaryon mass is estimated in CHIPS to be 200 MeV higher than in the bag model. The prediction for the mass of the α^* cluster is about the same in both models. It is close to $4 \cdot m_{\Delta}$.

PACS. 12.39.Ki Relativistic quark model – 12.40.Yx Hadron mass models and calculations

1 Introduction

The CHIPS event generator [1–3] is based on the phase space distribution for quarks. The u- and d-quarks are massless and all phase space integrals are simple. The formal assumption of the massless strange quark did not change the predicted spectra of hadrons, as the real mass of the strange quark is indirectly taken into account in the masses of the secondary strange hadrons.

It is not clear why low-energy processes, such as nuclear pion capture at rest [2] or photo-nuclear reactions below the pion production threshold [3], are successfully calculated on the quark level. To prove the applicability of the CHIPS model at low energies the masses of hadrons are calculated in this paper. When calculating hadronic masses it is not possible to neglect the mass of the strange quark. The mass of the strange quark ($m_s = 198\,\mathrm{MeV}$) is found to be comparable with the temperature value ($T_{\mathrm{c}} = 221\,\mathrm{MeV}$). The critical temperature was first proposed by R. Hagedorn [4], but for hadrons rather than for quarks. In quark models the critical temperature corresponds to the chiral-restoration temperature. The approach based on asymptotically free quarks seems to be reasonable because, as it was shown in [5], at temperatures close to the critical temperature the effective coupling constant $g(T)$ is small and it is possible to use the current masses of quarks and apply perturbative QCD.

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In this paper eq. (1) of the first publication of CHIPS [1] is explained in detail and generalized for the massive quarks. Only the light quark hadrons are calculated, although the phase space mass formula is general and can also be applied to hadrons consisting of heavier quarks. But the gluon exchange forces are so strong for heavy quarks that the kinematic contributions can be significantly distorted. So the light hadrons were selected because there exists an algorithm for calculating the color-electric and color-magnetic interactions. It should be noted that the color-electric and color-magnetic terms are not a part of the CHIPS model, and can be considered only as an external empirical part of the calculations.

The most informative section of the paper is the second section where the mean effective mass of a few quarks at constant temperature are calculated. In the third section the contributions of the color-electric and color-magnetic interactions are calculated. In the last section the masses of hadrons consisting of light quarks are calculated and compared both to experimental values and to the values calculated in the classic bag model [6].

2 The mean effective mass of a few quarks at critical temperature

The calculations in this section are general and can be considered as a kind of "thermodynamics" developed for the case of a small number of particles. In other words, this is an attempt to consider the state of a few labeled particles in the constant-temperature environment. In general, the energy fluctuations for the labeled particles are relatively big and the labeled particles can diffuse away from each other. In the case of colored particles the situation is more definite. Firstly, the colored quarks are confined, which means that they do not diffuse away from each other. Secondly, the colored quarks are combined in color singlets (hadrons), most of the ground states of which are stable. The large width of the energy distribution of the asymptotically free quarks is reduced because most of the decay channels into colored fragments are closed. Thirdly, the confined quark system can be considered as a relativistically invariant object at finite temperature.

The important question of the model is the nature of the local relativistic invariant environment at constant temperature. This question needs further clarification but just to have some idea the hypothesis of Nathan Isgur [7] can be mentioned. On the boundary of the perturbative and non-perturbative vacua the energy density of space becomes negative. At this condition a lot of virtual gluonic degrees of freedom can be excited on the hadronic surface, so this surface can play the role of the universal relativistic invariant "thermostat" (the environment at constant temperature). Alternatively, the T_{c} of the CHIPS model can be connected with the pion Mott temperature of the recent chiral quark model (CQM) [8] ($T_{\mathrm{c}}^{\mathrm{CQM}} = 186\,\mathrm{MeV}$).

For n-particles the integral over states with different total energies at a fixed temperature T_{c} (the Gibbs canonical distribution) can be written as

$$F_n(\beta) = \int e^{-\beta E} \Phi_n(E) \mathrm{d}^4 p, \tag{1}$$

where $\beta = \frac{1}{T_{\mathrm{c}}}$ is used just for convenience and $\Phi_n(E)$ is an n-quark phase space integral which is defined as

$$\Phi_n(E) = \int \delta\left(\sum_{i=1}^{n} E_i - E\right) \delta^3\left(\sum_{i=1}^{n} \mathbf{p}_i\right) \prod_{i=1}^{n} \frac{\mathrm{d}^3 \mathbf{p}_i}{2E_i}. \tag{2}$$

To make eq. (1) relativistically invariant it is necessary to rewrite it in the following form:

$$F_n(\beta) = \int_{(\Sigma m_i)^2}^{\infty} \mathrm{d}s \int \mathrm{d}^4 p\ \delta(p^2 - M^2)\ e^{-\beta E} \Phi_n(M), \tag{3}$$

where $s = M^2$ and M is the invariant mass of n-quarks. An integration over the energy (p_0) and the solid angle results in

$$F_n(\beta) = 2\pi \int_{(\Sigma m_i)^2}^{\infty} \mathrm{d}s \int_{M}^{\infty} \sqrt{E^2 - M^2} \mathrm{d}E e^{-\beta E}\ \Phi_n(M). \tag{4}$$

Using the McDonald's function K_1 one can rewrite this equation as

$$F_n(\beta) = \frac{4\pi}{\beta} \int_{\Sigma m_i}^{\infty} M^2\ K_1(\beta M)\ \Phi_n(M) \mathrm{d}M. \tag{5}$$

Substituting the integral from eq. (2) for $\Phi_n(E)$ in eq. (1), changing the order of integration and integrating over $\mathrm{d}^4 p$, one can obtain the following result:

$$F_n(\beta) = \prod_{i=1}^{n} \int_{m_i}^{\infty} e^{-\beta E_i} \frac{\mathrm{d}^3 p_i}{2E_i} = \prod_{i=1}^{n} f_i(\beta, m_i). \tag{6}$$

Using the same McDonald's function K_1, the f_i-functions can be written as

$$f_i(\beta, m_i) = 2\pi \frac{m_i}{\beta} K_1(\beta m_i). \tag{7}$$

With this notation the mean-squared invariant mass of the n-particles can be calculated as a ratio

$$\langle s_n \rangle = \frac{\int_{\Sigma m_i}^{\infty} M^4\ K_1(\beta M)\ \Phi_n(M) \mathrm{d}M}{\int_{\Sigma m_i}^{\infty} M^2\ K_1(\beta M) \Phi_n(M) \mathrm{d}M}. \tag{8}$$

Taking into account that the denominator is a second derivative of the numerator, eq. (8) can be rewritten in the following form:

$$\langle s_n \rangle = \frac{\mathrm{d}^2 \beta^2 F_n(\beta)}{F_n(\beta) \beta^2 (\mathrm{d}\beta)^2}. \tag{9}$$

To get the final result it is enough to substitute $f_n(\beta)$ from eq. (7) to eq. (6) and then calculate the derivatives. The result can be written as

$$\langle s_n \rangle = \sum_{i=1}^{n} m_i^2 + 2 \sum_{i=1}^{n} \sum_{j>i}^{n} (2T_{\mathrm{c}} + x_i)(2T_{\mathrm{c}} + x_j), \tag{10}$$

where $x_i = m_i \frac{K_0(m_i \beta)}{K_1(m_i \beta)}$. Taking into account an expansion of the McDonald's function for $m_i \gg T_{\mathrm{c}}$, we find that at this condition $x_i = m_i - \frac{1}{2} T_{\mathrm{c}}$, and eq. (10) gives the well-known thermodynamic result for $n \gg 1$: the mean kinetic energy per particle at temperature T_{c} is $\frac{3}{2} T_{\mathrm{c}}$. In addition it is found that for small n the mean kinetic energy can be calculated more accurately as $\frac{3(n-1)}{2n} T_{\mathrm{c}}$. Equation (10) for the case of massless particles ($x_i = 0$) proves eq. (1) of [1].

For hadrons all channels of decay in colored partons are closed. It makes hadrons narrow or even stable. When calculating the hadronic masses it was assumed that, when hadrons become narrow, the value of the mean-squared mass is not changed. To clarify why the mass formula, eq. (10), is more promising for the mass spectrum calculations than a sum of the constituent masses of quarks, let us consider the simplest case of π/ω mesons and N/Δ baryons. For massless u- and d-quarks the color-electric contribution, as shown in [6], is zero. So only the color-magnetic interactions split the π/ω mesons and N/Δ baryons. As explained in the following section the unsplit masses are $m_M = \frac{1}{4}(3 \cdot m_\omega + m_\pi) = 622\,\mathrm{MeV}$ and $m_B = \frac{1}{2}(m_\Delta + m_N) = 1085.5\,\mathrm{MeV}$. The ratio is 1.745, not $1.5 = \frac{3 \cdot m_q}{2 \cdot m_q}$. The bag model in the case of massless u- and d-quarks gives 1.65. Equation (10) gives 1.732, which is much closer to the experimental value.

3 Color-electric shift and color-magnetic splitting of hadronic masses

As shown in [6], for the light hadrons the color-electric term is zero for hadrons with equal masses of quarks (π, ω, ϕ, N, Δ, Ω), because in this case the color charge density vanishes locally. For hadrons with different masses of quarks (K, Λ, Σ, Ξ) the color-electric shift is a constant value, because in the K-meson the contribution of only one qs electrostatic interaction must be added and in any baryon two qs interactions must be added, but each is twice as small as in the K-meson, since the colored-gluon interaction is proportional to $\langle\lambda_i,\lambda_j\rangle = -\frac{\langle\lambda^2\rangle}{n-1}$ (it is $-\frac{16}{3}$ for mesons and $-\frac{8}{3}$ for baryons).

The color-magnetic splitting is more complicated. Using the $\delta(r)$-term of the spin-spin interaction part in the Hamiltonian, and by replacing the masses of the quarks by their mean energies one can show that in addition to the same factor $\langle\lambda_i,\lambda_j\rangle$ the color-magnetic contribution to masses is proportional to the spin product $\langle s_i,s_j\rangle$, inversely proportional to the product of the mean energies of the quarks $E_i \cdot E_j$, and proportional to $|\psi(0)|^2$. The $|\psi(0)|^2$ factor is inversely proportional to the perturbative volume of the hadron.

In the case of equal mean energies of quarks the sum of the spin products can be easily calculated:

$$n \cdot \langle s^2\rangle + 2 \cdot \sum_{i>j}\langle s_i,s_j\rangle = \langle S^2\rangle, \qquad (11)$$

where n is the number of quarks and $\langle s^2\rangle = s\cdot(s+1)$ is $\frac{3}{4}$ for quarks and octet baryons, $\frac{15}{4}$ for decuplet baryons, 0 for pseudo-scalar mesons, and 2 for vector mesons. So for the pseudo-scalar mesons the sum of products is $-\frac{3}{4}$ and for the vector mesons it is $\frac{1}{4}$. Similarly, it is $-\frac{3}{4}$ for the octet baryons and $\frac{3}{4}$ for the decuplet baryons. These coefficients were used in the previous section, where the unsplit π/ω and N/Δ masses were evaluated.

If the masses of quarks are different and the $\frac{1}{E_i\cdot E_j}$ factor cannot be factored out, it is necessary to know in which state the quarks with the same masses are. In the case of the qqs hadrons the qq pair can be in the $S=0$ or $S=1$ state. As the wave function is anti-symmetric with respect to the color charge of quarks, the identical quarks cannot be in the $S=0$ state, so only the ud-diquark can be in the $S=0$ state. The residual s-quark cannot have spin-spin interactions with the $S=0$ ud-diquark, so both qs contributions are 0. To match the total sum the $\langle s_u,s_d\rangle$ spin product must be equal to $-\frac{3}{4}$. In the opposite case, when uu-, ud-, or dd-diquarks are in the $S=1$ state, the spin product for such qq pairs is $\frac{1}{4}$, and the qs spin products can be found from the total sum value: $\langle s_q,s_s\rangle = -\frac{1}{2}$. The $a_{ij} = -\langle\lambda_i,\lambda_j\rangle\langle s_i,s_j\rangle$ coefficients together with the b-flags of the color-electric contribution are listed in table 1. The mean energy for equal masses of quarks is $\frac{\sqrt{\langle s_n\rangle}}{n}$. For K-mesons the mean energies are calculated using the condition of equal moments of quarks. For the Λ- and Σ-hyperons the mass excess in respect to the N/Δ unsplit mass is added to the energy of the strange quark. For the Ξ-hyperons the missing mass with respect to the unsplit Ω^- mass is subtracted from the mean energy of the u- or d-quark.

Table 1. Coefficients of the color-magnetic and flags of the color-electric contributions to hadronic masses.

H	a_{qq}	a_{qs}	a_{ss}	b
π^0	-4	0	0	0
ω	$\frac{4}{3}$	0	0	0
K	0	-4	0	1
K^*	0	$\frac{4}{3}$	0	1
η_8	0	0	-4	0
ϕ	0	0	$\frac{4}{3}$	0
d^*	$-\frac{4}{3}$	0	0	0
α^*	-3	0	0	0
N	-2	0	0	0
Δ	2	0	0	0
Λ	-2	0	0	1
Σ	$\frac{2}{3}$	$-\frac{8}{3}$	0	1
Σ^*	$\frac{2}{3}$	$\frac{4}{3}$	0	1
Ξ	0	$-\frac{8}{3}$	$\frac{2}{3}$	1
Ξ^*	0	$\frac{4}{3}$	$\frac{2}{3}$	1
Ω^-	0	0	2	0

The volume of the perturbative space was assumed to be proportional to $\sqrt{n}$. Thus, the resulting formula for the hadronic masses can be written as

$$M = \sqrt{\langle s_n\rangle} + b\cdot E_{\mathrm{CE}} + \frac{A_{\mathrm{CM}}}{\sqrt{n}}\sum_{i=1}^{n}\sum_{j<i}\frac{a_{ij}}{E_i\cdot E_j}, \qquad (12)$$

where $\langle s_n\rangle$ is defined by eq. (10), and the b and a_{ij} coefficients are listed in table 1.

4 Comparison with experimental data

The masses of hadrons have not been fitted. Instead, the main parameters were estimated using different groups of hadrons. The T_{c} and A_{CM} parameters were estimated using the masses of π/ω and N/Δ hadrons which consist of only u- and d-quarks and are not shifted by the color-electric interactions. The masses of the ϕ and Ω^- hadrons, which are also not shifted by the color-electric interactions, were used to estimate m_s. The color-electric constant E_{CE} was estimated to fit the rest of the strange mesons and hyperons. The small mass of the d-quark was used to find out how the isotopic shifts of hadronic masses are correlated with the mass difference of the u- and d-quarks. The calculated masses are listed in table 2 for $T_{\mathrm{c}} = 221\,\mathrm{MeV}$, $m_u = 0$, $m_d = 2.7\,\mathrm{MeV}$, $m_s = 198\,\mathrm{MeV}$, $E_{\mathrm{CE}} = 16\,\mathrm{MeV}$, $A_{\mathrm{CM}} = 0.0165\,\mathrm{GeV}^3$. These parameters match the $B^{1/4}$, Z_0, α_s, m_q, and m_s parameters of the two

Table 2. Masses of hadrons. The M^{bag} values are taken from [6] for $m_q = 0$, $m_s = 279$ MeV and $m_q = 108$ MeV, $m_s = 353$ MeV cases. In the CHIPS calculations $T_c = 221$ MeV, $m_u = 0$, $m_d = 2.7$ MeV, $m_s = 198$ MeV, $E_{\text{CE}} = 16$ MeV, $A_{\text{CM}} = 0.0165\,\text{GeV}^3$. All hadronic masses are quoted in MeV.

H	M^{exp}	$M^{\text{bag}}_{\text{cur}}$	$\lvert\Delta M\rvert$	$M^{\text{bag}}_{\text{con}}$	$\lvert\Delta M\rvert$	$M^{T_c}_{\text{cur}}$	$\lvert\Delta M\rvert$
π^0	140	280	140	175	35	152	12
ω	783	783	0	783	0	785	2
p	938	938	0	938	0	939	1
n	939	938	1	938	1	941	2
Δ	1232	1233	1	1233	1	1231	1
K^+	494	497	3	371	123	485	9
K^0	498	497	1	371	127	489	9
K^{*+}	892	928	36	925	33	898	6
K^{*0}	896	928	32	925	29	899	3
Λ	1116	1105	11	1103	13	1123	7
Σ^+	1189	1144	45	1145	44	1182	7
Σ^0	1193	1144	49	1145	48	1185	8
Σ^-	1197	1144	53	1145	52	1187	10
Σ^{*+}	1383	1382	1	1381	2	1382	1
Σ^{*0}	1384	1382	2	1381	3	1384	0
Σ^{*-}	1387	1382	5	1381	6	1385	2
η_8	754	693	61	693	61	742	12
ϕ	1019	1068	49	1063	44	1018	1
Ξ^0	1315	1289	26	1286	29	1320	5
Ξ^-	1321	1289	32	1286	35	1323	2
Ξ^{*0}	1532	1529	3	1528	4	1531	1
Ξ^{*-}	1535	1529	6	1528	7	1533	2
Ω^-	1673	1672	1	1672	1	1674	1
d^*		2160				2366	
α^*		4880				4823	

cases of calculations in [6] (the "cur" index corresponds to the current masses of quarks $m_q = 0$, $m_s = 279$ MeV and the "con" index corresponds to the constituent masses of quarks $m_q = 108$ MeV, $m_s = 353$ MeV).

Comparison of the two models with experiment shows that the mass values calculated in the CHIPS model are much closer to the experimental values. For $m_q = 0$ the bag model gives a very bad prediction for the pion mass. For $m_q = 108$ MeV the bag model prediction for m_π is better, but in this case the kaon mass becomes too small. All the other mass values in the bag model seem to be independent of the u- and d-quark masses. Another important value is the Λ/Σ mass difference which is experimentally 80.6 MeV. In the bag model it is about 40 MeV and in CHIPS it is 59 MeV. Another test can be done for the $m_{\Sigma^*} - m_\Sigma$ and $m_{\Xi^*} - m_\Xi$ values which must be equal in $SU(6)$ symmetric models. The mean experimental value for these two differences is 203.5 MeV. In the bag model it is 239 MeV and in the CHIPS model it is 205.1 MeV. The last test is the ϕ-meson mass, which is much closer to the experimental value in the CHIPS calculations than in the bag model calculations.

One line in the table needs special explanation. This is the η_8 mass. It is not clear how the experimental value of the η_8-particle can be estimated, because the annihilation diagrams [6] make additional contribution to the η masses. The result of this mixing can be written for the two-quark states $\eta = \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s)$ and $\eta' = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s)$ in the form of the mass matrix

$$m_\eta = \begin{pmatrix} m_\pi + \frac{2}{3}E_{gg}\,, & \frac{\sqrt{2}}{3}E_{gg} \\ \frac{\sqrt{2}}{3}E_{gg}\,, & m_{\eta_8} + \frac{1}{3}E_{gg} \end{pmatrix}, \tag{13}$$

where m_π is used for m_{η_0}. The resulting mass formula is

$$m_{\eta_8} = m_{\eta'} - \frac{1}{2}(m_\eta - m_\pi). \tag{14}$$

This is how the "experimental" value of the m_{η_8} was calculated. Another interpretation of the η and η' masses could be the splitting of the m_{η_8} because of the annihilation diagrams. In this case one might expect that $m_{\eta_8} = \frac{1}{2}(m_{\eta'} + m_\eta)$. It is interesting that both approaches give the same value for m_{η_8} (754.0 MeV and 752.5 MeV correspondingly), so the additional mass formula $m_\eta = \frac{1}{2}(m_{\eta'} + m_\pi)$ can be written. The physical meaning of this empirical mass formula is not clear. Nevertheless, the CHIPS value coincides with the m_{η_8} mass with good accuracy.

The calculated isotopic shifts of hadrons determined by the mass difference of the d- and u-quarks are qualitatively correct, but in the case of Σ and Σ^* the mass of Σ^- differs from the mass of Σ^0 by more than the mass of Σ^0 differs from the mass of Σ^+. Taking into account that, being proportional to the sum of the charge products of the quarks, the electromagnetic shift for the positive strange hyperons is about zero, the negative hyperons are shifted up and the neutral hyperons are shifted down by the same value, the residual electromagnetic corrections can be estimated as 1 MeV (0.8 MeV for the Σ-hyperons and 1.3 MeV for the Σ^*-hyperons).

The mass of the lowest dibaryon state ($m_d^* = 2366$ MeV) calculated in CHIPS is larger than the mass predicted by the bag model (2160 MeV). It better matches the maximum in the NN interaction cross-section ($\sqrt{s} \approx 2500$ MeV). So one can expect the dibaryons to play an important role in nucleon-nucleon interactions. The mass of the biggest S-wave multiquark cluster (α^*) is almost the same for both models. It is close to the total mass of the four Δ-isobars.

5 Conclusion

The CHIPS model based on the "temperature of the non-perturbative vacuum" hypothesis seems to be at least as successful as the bag model based on the "pressure of the non-perturbative vacuum" hypothesis. In both models the current masses of quarks can be used.

The d^* and α^* clusters are considered to have a common perturbative space, but they must not be identified with the nuclear clusters of the CHIPS event generator [2], as the nuclear clusters result from the quark exchange interactions between nuclear nucleons and the d^* and α^* clusters are just hypothetical particles.

The CHIPS model is $SU(3) \times SU(6)$ symmetric and the success of the CHIPS calculations of the masses of hadrons consisting of light quarks proves the applicability of the CHIPS event generator at extremely low energies, when the free energy is small and a large number of degrees of freedom is not explicit.

It is a pleasure to thank Prof. Yu. A. Simonov and Dr. M. Sang for useful discussions. This work was partially supported by the grant CERN-INTAS 99-0377.

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